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A FEM BASED APPROACH TO FIND THE STIFFNESS MATRIX, GLOBAL CO-EFFICIENT MATRIX AND NODE VOLTAGES OF WAVEGUIDE IN DIFFERENT MODELS

Chandrashekar.K*, Girish V. Attimarad
* Dept. of Inst. Tech., P.D. Institute of Technology, Hospet, India
Dept of E&CE, Dayanandasagar College of Engineering, Bangalore, India

ABSTRACT
The Paper discusses the usage of Finite element method (FEM) in order to find triangular element Stiffness, Global co-efficient matrices of each model and calculate all node voltages by using Band matrix method or Iterative method of different models Viz. Pipe, Step, Bend and Tapering model type waveguide discontinuities. Finally, stiffness, global co-efficient matrices and all node voltages are tabulated as shown.

KEYWORDS: Finite Element Method (FEM), Stiffness Matrix, Global co-efficient matrix and Node voltage, Band matrix method or Iterative method, Pipe, Step, Bend and taper type waveguide, Discontinuities, Laplace equations.

INTRODUCTION
Discontinuities in waveguides can be defined as defects of waveguide structure by two or more factious boundaries that occur inevitably in many waveguide systems due to construction tolerance and misalignment in component intersection [1] [2]FEM method is applied to the analysis of discontinuities in waveguides of different models and compared with early work with models like pipe and step waveguide discontinuities [3].Numerical methods like finite difference method (FDM), Finite element method (FEM) and Markov chain method (MCM) are used to solve two dimensional method steady heat flow problem with Dirichchlet Boundary condition [5].A powerful technique for fast generation of sparse systems of linear equations arising in computational electromagnetic in a finite element method using higher order elements, to obtain a graphics processing unit(GPU) for both numerical integration and matrix assembly[6].

MATERIALS AND METHODS
Finite element Method
FEM was originally used only for the analysis of waveguide structures [3]. The earlier mathematical treatment was proposed in 1943 by Courant [1]. However, FEM was not used for electromagnetic propagation problems until 1968. Since then, this method has been employed to solve the difficulties faced by waveguides, electric machines and to analyze semiconductor devices, micro strips and absorption of EM radiation by biological bodies, etc. The FEM of any problem involves four steps [10]:

- Discretizing the solution region into a finite number of subdivisions or elements.
- Deriving governing equations for a typical element.
- Assembling of all elements in the solution region.
- Solving the system of equations obtained.

Finite Element Discretization
Discretization is a process in which the solution region is divided into sub domains called finite elements. The two-dimensional solution region is shown in Figure 1(a). The solution region is divided into a number of finite elements called a sub-region, as shown in Figure 1(b), and $V(x, y)$ is the potential distribution. In Figure 1(b), the solution region is subdivided into nine non-overlapping finite elements, elements 6, 8, and 9 are four-node quadrilaterals and the other elements are three-node triangles. For easy computation, all elements should be of same shape, i.e., triangle

with three-nodes. Each quadrilateral element can be further divided into two triangles so that there are 12 triangular elements altogether.

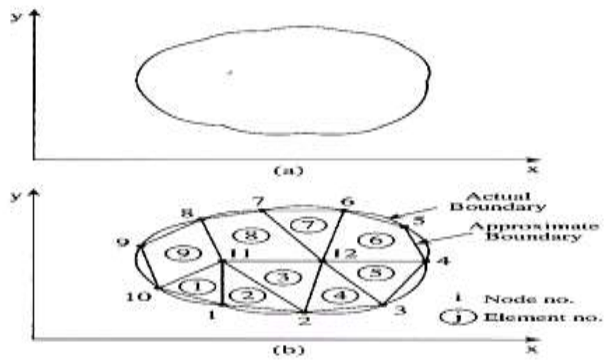


Fig. 1. (a) Solution region. (b) Finite discretization

The approximate potential distribution for the whole region is given by

$$V(x,y) \cong \sum_{e=1}^{N} V_e(x,y) \tag{1}$$

Where N is the number of triangular elements into which the solution region is divided. The most common form of approximation for Ve within an triangular element is polynomial and is given by

$$V_e(x,y) = a + bx + cy \tag{2}$$

For a quadrilateral element, the potential distribution is given by

$$V_e(x,y) = a + bx + cy + d \tag{3}$$

The potential Ve in general is a non-zero within element. Quadrilateral elements are not suitable for curved boundaries when compared to triangular elements. In this study, triangular elements have been used throughout the analysis. It should be noted that the assumption of linear variation of potential within the triangular element as in (1) is the same as assuming that the electric field is uniform within the element and is given by

$$E_e = -\nabla V_e = -(ba_x + ca_y) \tag{4}$$

Governing Equations for Each Node in Triangular Elements

Consider a typical triangular element as shown in Figure (2). The potential V_{e_1}, V_{e_2} and V_{e_3},at nodes 1, 2, and 3 respectively, were obtained using equation (5):

$$\begin{bmatrix} V_{e_1} \\ V_{e_2} \\ V_{e_3} \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{5}$$

The coefficients a, b, and c were determined from equation (6):

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} V_{e_1} \\ V_{e_2} \\ V_{e_3} \end{bmatrix} \tag{6}$$

From equation (2), the following equation can be derived:

$$V_e = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} (x_2y_3 - x_3y_2) & (x_3y_1 - x_1y_3) & (x_1y_2 - x_2y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \begin{bmatrix} V_{e_1} \\ V_{e_2} \\ V_{e_3} \end{bmatrix} \tag{7}$$

Where

$$\alpha_1 = \frac{1}{2A}\left[(x_2y_3 - x_3y_2) + (y_2 - y_3) + (x_3 - x_2)y\right] \tag{8}$$

$$\alpha_2 = \frac{1}{2A}\left[(x_3y_1 - x_1y_3) + (y_3 - y_1)x + (x_1 - x_3)y\right] \tag{9}$$

$$\alpha_3 = \frac{1}{2A}\left[(x_1y_2 - x_2y_1) + (y_1 - y_2)x + (x_2 - x_1)y\right] \tag{10}$$

A is the area of the element, and α1, α2, and α3 are element shape functions.

$$2A = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\left[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)\right] \tag{11}$$

$$\alpha_i(X_j, Y_j) = \begin{cases} 1, & i = j \\ 0 & i \neq j \end{cases} \tag{12}$$

$$\begin{bmatrix} c_{11}^e & c_{12}^e & c_{13}^e \\ c_{21}^e & c_{22}^e & c_{23}^e \\ c_{31}^e & c_{32}^e & c_{33}^e \end{bmatrix} = \left[C^e\right] \tag{13}$$

$$P_1 = y_2 - y_1 \qquad Q_1 = x_3 - x_2$$
$$P_2 = y_3 - y_1 \qquad Q_2 = x_1 - x_3$$
$$P_3 = y_1 - y_2 \qquad Q_3 = x_2 - x_1 \tag{14}$$

$$c_{ij}^e = \frac{1}{4A}(P_i P_j + Q_i Q_j) \quad \text{and} \quad A = \frac{1}{2}(P_2 Q_3 + P_3 Q_2) \tag{15}$$

Assembling All Nodes of the Element

All typical elements in the solution region were assembled. The energy associated with the assemblage of all the elements in the mesh is given by

$$W = \sum_{e=1}^{N} W_e = \frac{1}{2}\varepsilon[V]^t[C][V] \tag{16}$$

Where
$$[V] = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_n \end{bmatrix}$$

and n is the number of nodes; N is the number of elements; and [C] is called the overall coefficient matrix, which is the assemblage of individual element coefficient matrices.
Solving the Resulting Equations

The resulting equations are solved by the band matrix method. After numbering all free nodes, from the first node, and the fixed nodes, the equation can be written as

$$W = \frac{1}{2}\varepsilon\begin{bmatrix} V_f & V_p \end{bmatrix}\begin{bmatrix} c_{ff} & c_{fp} \\ c_{pf} & c_{pp} \end{bmatrix}\begin{bmatrix} V_f \\ V_p \end{bmatrix} \tag{17}$$

where subscripts f and p refer to the free nodes and fixed potential, respectively. Since Vp is constant, differentiation was performed only with respect to Vf we get:

$$\left[c_{ff}\right]\left[c_p\right] = -\left[c_{fp_p}\right]\left[V_p\right] \tag{18}$$

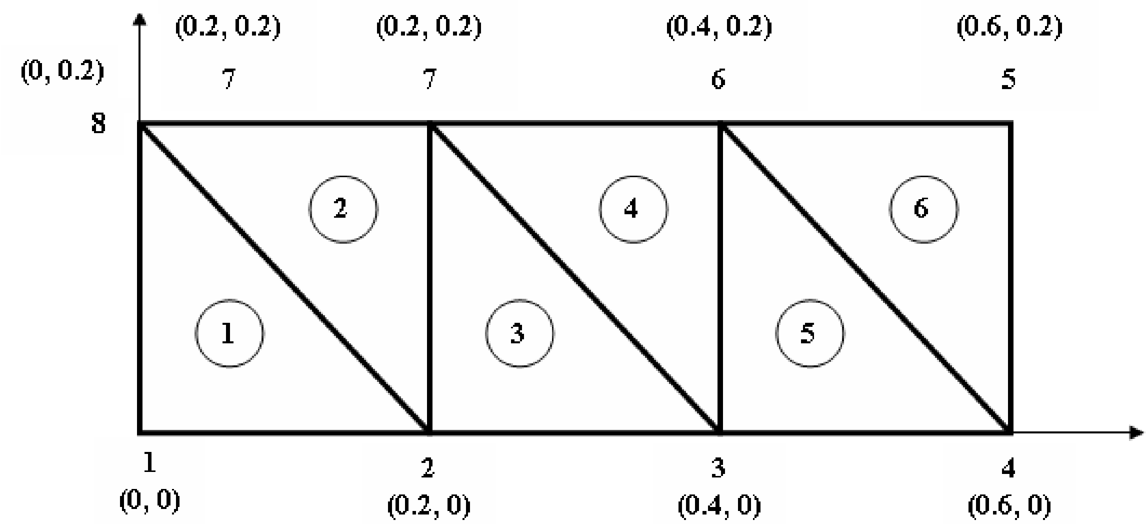
This can be written as

$$[A]\,[V]=[B], \quad (19)$$

$$[A]\,=[V]^{-1}[B], \quad (20)$$

Where [V]= [Vf], [Cff] and [B]=-[Cff] [Vp]. (21)

Since [A] is a matrix, the potential at the free nodes can be found by using equation (20).



Pipe Type Mode

Table 1. Stiffness matrix of each element of pipe type model

Element	Element Coefficient Matrix			Element	Element Coefficient Matrix		
1	1	0	-0.5	4	0.5	-0.5	0
	0	0.5	-0.5		-0.5	1	-0.5
	-0.5	-0.5	1		0	-0.5	0.5
2	0.5	-0.5	0	5	1	0	-0.5
	-0.5	1	-0.5		0	1	-0.5
	0	-0.5	0.5		-0.5	-0.5	0.5
3	1	-0.5	-0.5	6	-0.5	-0.5	0
	0	0.5	0		-0.5	1	0
	-0.5	0	0.5		0	-0.5	0.5

Table 2. Node Voltage of each node of pipe type

1	0	0	0
2	0.2	0	0
3	0.4	0	10
4	0.6	0	20
5	0	0.2	0
6	0.2	0.2	20
7	0.4	0.2	10
8	0.6	0.2	0

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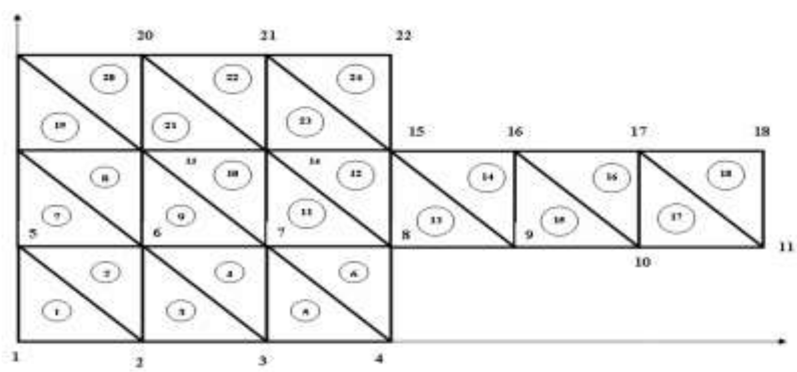
Table 3. Global co-efficient matrix of pipe type model

	1.	2.	3.	4.	5.	6.	7.	8.
1.	1	0	0	0	0	0	0	0.5
2.	0	1	-0.5	0	0	0	0	1
3.	0	-0.5	2	0	0	-1	0	0
4.	0	0	0	1.5	-0.5	0.5	0	0
5.	0	0	0	-0.5	1		0	0
6.	0	0	-1	0.5	0	2	0.5	0
7.	0	0	0	0	0	0.5	1	-0.5
8.	0.5	1	0	0	0	0	-0.5	1.5



Step Type Model

Table 4. Stiffness matrix of each element of step type mod

Element	Element Matrix Coefficient			Element	Element Matrix Coefficient			Element	Element Matrix Coefficient		
1	1	-0.5	-0.5	9	1	-0.5	-0.5	17	1	-0.5	-0.5
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
2	0.5	-0.5	0	10	0.5	-0.5	0	18	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	-0.5		0	-0.5	-0.5		0	-0.5	-0.5
3	1	-0.5	-0.5	11	1	-0.5	-0.5	19	1	-0.5	-0.5
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
4	0.5	-0.5	0	12	0.5	-0.5	0	20	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	-0.5		0	-0.5	-0.5		0	-0.5	-0.5
5	1	-0.5	-0.5	13	1	-0.5	-0.5	21	1	-0.5	-0.5
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
6	0.5	-0.5	0	14	0.5	-0.5	0	22	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	-0.5		0	-0.5	-0.5		0	-0.5	-0.5
7	1	-0.5	-0.5	15	1	-0.5	-0.5	23	1	-0.5	-0.5
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5

8	0.5	-0.5	0	16	0.5	-0.5	0	24	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	-0.5		0	-0.5	-0.5		0	-0.5	-0.5

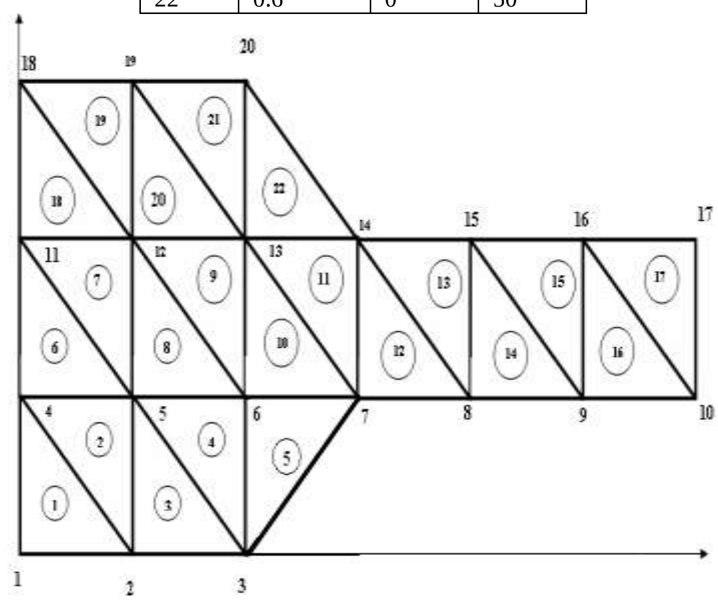
Table 5. Global co-efficient matrix of step type model

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	1	-0.5	-0.5	0	0	0	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-0.5	2	-0.5	0	0	0	0	-0.5	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-0.5	0.5	2	-0.5	0	0	0	0	-0.5	-0.5	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	-0.5	1	0	0	0	0	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	2	-0.5	0	0	0	0	0.5	0	0	0	-0.5	-0.5	0	0	0	0	0	0
6	0	0	0	0	-0.5	2	-0.5	0	0	0	0	0	0	0	0	-0.5	-0.5	0	0	0	0	0
7	0	0	0	0	0	-0.5	1	0	0	0	0	0	0	0	0	0	-0.5	0	0	0	0	0
8	-0.5	-0.5	0	0	0	0	0	2.5	-1	0	0	-0.5	0	0	0	0	0	0	0	0	0	0
9	0	-0.5	-0.5	0	0	0	0	-1	3	-1	0	-0.5	0.5	0	0	0	0	0	0	0	0	0
10	0	0	-0.5	-0.5	0	0	0	0	-1	3	0	0	0	-0.5	0	0	0	0	0	0	0	0
11	0	0	0	0	-0.5	0	0	0	0	-1	2.5	0	0	-0.5	-0.5	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	-0.5	-0.5	0	0	2.5	-1	0	0	0	0	0	-0.5	0	0	0
13	0	0	0	0	0	0	0	0	-0.5	-0.5	0	-1	4	-1	0	0	0	0	-0.5	-0.5	0	0
14	0	0	0	0	0	0	0	0	-0.5	0	-0.5	0	-1	4	-1	0	0	0	0	-0.5	-0.5	0
15	0	0	0	0	-0.5	0	0	0	0	0	-0.5	0	0	-1	3	1	0	0	0	0	-0.5	-0.5
16	0	0	0	0	-0.5	-0.5	0	0	0	0	0	0	0	0	-0.5	2	-0.5	0	0	0	0	0
17	0	0	0	0	0	-0.5	1	0	0	0	0	0	0	0	0	-0.5	2	-0.5	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	0	0.5	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	-0.5	-0.5	0	0	0	0	0	1.5	-0.5	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	1	-0.5	0	0	0	0	-0.5	2	-0.5	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	-0.5	0	0	0	0	-0.5	2	-0.5
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.5

Table 6. Node Voltage of each node of step type model

1	0	0	0
2	0.2	0	0
3	0.4	0	0
4	0.6	0.2	100
5	0	0.2	50
6	0.2	0.2	25

7	0.4	0.2	0
8	0.6	0.2	0
9	0.8	0.2	25
10	1	0.2	0
11	1.2	0.4	0
12	0	0.4	50
13	0.2	0.4	25
14	0.4	0.4	0
15	0.6	0.4	0
16	0.8	0.4	50
17	1	0.4	100
18	1.2	0.6	0
19	0	0.6	25
20	0.2	0.6	0
21	0.4	0.6	100
22	0.6	0	50



Taper Type Model

Table 7. Stiffness matrix of each element of taper type model

Element	Element Matrix Coefficient			Element	Element Matrix Coefficient			Element	Element Matrix Coefficient		
	1	-0.5	-0.5		1	-0.5	-0.5		1	-0.5	-0.5
1	-0.5	0.5	0	9	-0.5	0.5	0	17	-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
	0.5	-0.5	0		0.5	-0.5	0		0.5	-0.5	0
2	-0.5	1	-0.5	10	-0.5	1	-0.5	18	-0.5	1	-0.5
	0	-0.5	0.5		0	-0.5	0.5		0	-0.5	0.5
	1	-0.5	-0.5		1	-0.5	-0.5		1	-0.5	-0.5
3	-0.5	0.5	0	11	-0.5	0.5	0	19	-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
	0.5	-0.5	0		0.5	-0.5	0		0.5	-0.5	0
4	-0.5	1	-0.5	12	-0.5	1	-0.5	20	-0.5	1	-0.5
	0	-0.5	0.5		0	-0.5	0.5		0	-0.5	0.5
	1	-0.5	-0.5		1	-0.5	-0.5		1	-0.5	-0.5
5	-0.5	0.5	0	13	-0.5	0.5	0	21	-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
6	0.5	-0.5	0	14	0.5	-0.5	0	22	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5

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	0	-0.5	0.5		0	-0.5	0.5		0	-0.5	0.5
	1	-0.5	-0.5		1	-0.5	-0.5				
7	-0.5	0.5	0	15	-0.5	0.5	0				
	-0.5	0	0.5		-0.5	0	0.5				
	0.5	-0.5	0		0.5	-0.5	0				
8	-0.5	1	-0.5	16	-0.5	1	-0.5				
	0	-0.5	0.5		0	-0.5	0.5				

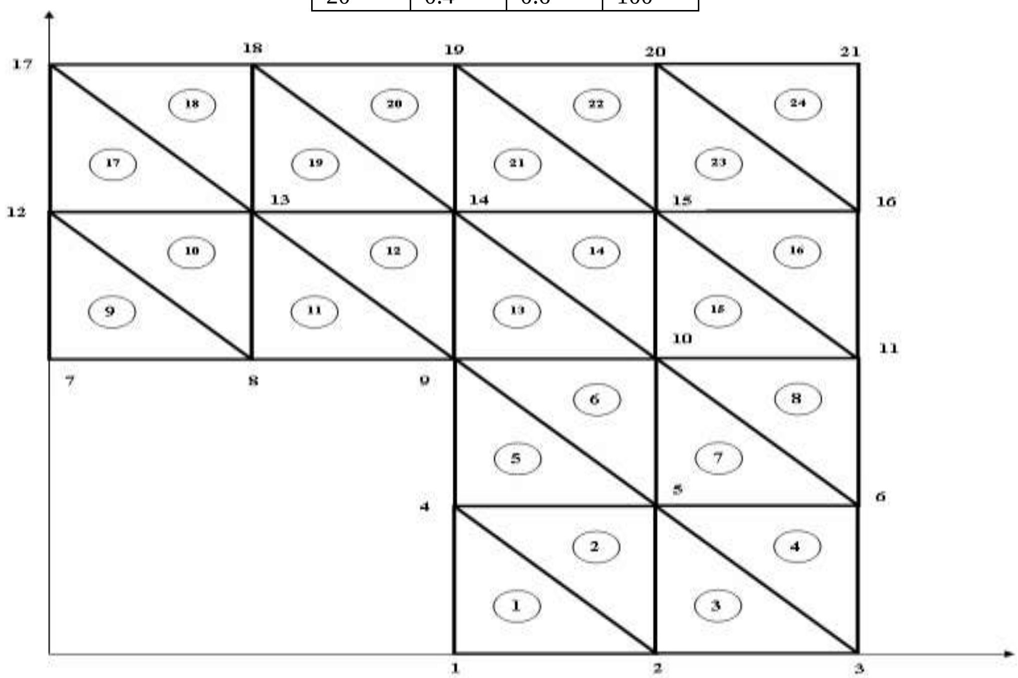
Table 8. Global co-efficient matrix of taper type model

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1	-0.5	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-0.5	2	-0.5	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	-0.5	1	0	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	-0.5	0	0	-0.5	-1	0	0	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0
5	0	-1	0	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	-0.5	0	0	2	0	0	0	0	-0.5	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	-0.5	0	0	0	-0.5	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	-0.5	2	-0.5	0	0	0	1	0	0	0	0	0	0	0	0
9	0	0	0	-0.5	0	0	0	-0.5	3.5	-1	0	0	0	-1	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	-1	3	-1	0	0	0	-0.5	0	0	0	0	0	0
11	0	0	0	0	0	-0.5	0	0	0	-1	2	0	0	0	-0.5	-0.5	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	-0.5	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	-1	3	-1	0	0	0	0	-1	0	0
14	0	0	0	0	0	0	0	0	-1	0	0	0	-1	3	-1	0	0	0	-0.5	0	0
15	0	0	0	0	0	0	0	0	0	-1.5	-0.5	0	0	-1	3	-1	0	0	0	-1	0
16	0	0	0	0	0	0	0	0	0	0	-0.5	0	0	0	-1	2	0	0	0	0.5	0.5
17	0	0	0	0	0	0	0	0	0	0	0	-0.5	0	0	0	0	1	-0.5	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	2	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0	0	0	2	0.5	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0.5	0	0	0.5	1.5	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	1

Table 9. Node Voltage of each node of taper type model

1	0	0	0
2	0.2	0	0
3	0.4	0	50
4	0	0.2	0
5	0.2	0.2	25
6	0.4	0.2	50
7	0.6	0.2	0

8	0.8	0.2	0
9	1	0.2	0
10	1.2	0.2	0
11	0	0.4	50
12	0.2	0.4	100
13	0.4	0.4	0
14	0.6	0.4	0
15	0.8	0.4	0
16	1	0.4	0
17	1.2	0.4	25
18	0	0.6	0
19	0.2	0.6	50
20	0.4	0.6	100



Bend Type Model

Table10. Stiffness matrix of each element of Bend type model

Element	Element Matrix	Coefficient		Element	Element Matrix	Coefficient		Element	Element Matrix	Coefficient	
1	1	-0.5	-.05	9	1	-0.5	-0.5	17	1	-0.5	-.05
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
2	0.5	-0.5	0	10	0.5	-0.5	0	18	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	-0.5		0	-0.5	0.5		0	-0.5	-0.5
3	1	-0.5	-0.5	11	1	-0.5	-0.5	19	1	-0.5	-.05
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
4	0.5	-0.5	0	12	0.5	-0.5	0	20	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	0.5		0	-0.5	-0.5		0	-0.5	-0.5
5	1	-0.5	-0.5	13	1	-0.5	-.05	21	1	-0.5	-.05
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
6	0.5	-0.5	0	14	0.5	-0.5	0	22	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5

	0	-0.5	0.5		0	-0.5	-0.5		0	-0.5	-0.5
7	1	-0.5	-0.5	15	1	-0.5	-.05	23	1	-0.5	-.05
	-0.5	0.5	0		-0.5	0.5	0		-0.5	0.5	0
	-0.5	0	0.5		-0.5	0	0.5		-0.5	0	0.5
8	0.5	-0.5	0	16	0.5	-0.5	0	24	0.5	-0.5	0
	-0.5	1	-0.5		-0.5	1	-0.5		-0.5	1	-0.5
	0	-0.5	0.5		0	-0.5	-0.5		0	-0.5	-0.5

Table11. Global co-efficient matrix of Bend type model

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	-0.5	0	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-0.5	2	-0.5	-0.5	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	-0.5	1.5	0	-0.5	-0.5	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
4	-0.5	-0.5	0	2	0	0	0	0	0	0	-0.5	0	0	0	0	0	0	0	0	0
5	0	-0.5	0.5	0	2	1	0	0	0	0	-0.5	0.5	0	0	0	0	0	0	0	0
6	0	0	-0.5	0	1.5	2	-1	0	0	0	0	-0.5	0	0	0	0	0	0	0	0
7	0	0	-0.5	0	0	-1	2.5	0	0	0	0	0	-1	-0.5	0	0	0	0	0	0
8	0	0	0	0	0	0	-0.5	2	0	0	0	0	0	-0.5	-0.5	0	0	0	0	0
9	0	0	0	0	0	0	0	0	2	-0.5	0	0	0	0	-0.5	-0.5	0	0	0	0
10	0	0	0	0	0	0	0	0	-0.5	1	0	0	0	0	0	-0.5	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0.5	-1	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	-1	3	1	0	0	0	0	-0.5	-0.5	0
13	0	0	0	0	0	0	0	0	0	0	0	-1	3	-0.5	0	0	0	0	-1	-0.5
14	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	2.5	-0.5	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	2	-0.5	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	2	0.5	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	0.5	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.5	0.5	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	2	-0.5
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.5	1

Table12. Node Voltage of each node of Bend type model

1	0	0	0
2	0.2	0	0
3	0.4	0	0
4	0.6	0	0
5	0.8	0	0
6	1	0	50
7	0	0.2	0
8	0.2	0.2	18.18
9	0.4	0.2	36.36
10	0.6	0.2	59.09
11	0.8	0.2	100
12	0	0.4	0
13	0.2	0.4	36.36
14	0.4	0.4	68.18
15	0.6	0.4	100
16	0	0.6	0
17	0.2	0.6	59.09
18	0.4	0.6	100
19	0	0.8	0
20	0.2	0.8	100
21	0	1	50

RESULTS AND DISCUSSION

Finite element method is presented to obtain scattering matrix (reflection and transmission co-efficient), solving the problem in waveguide junction (E-Plane and H-Plane), filter design and stiffness matrix, global co-efficient matrix and node potential of each node of waveguide model.

Figures shows Pipe type model, Step type model, Taper type model and Bend type model. Finite element method is presented to obtain each model stiffness matrix of each triangular element; global co-efficient matrix and node potential are tabulated.

Table13

Model No	Type model	No of triangular elements	Nodes	Matrix/ node voltage	Table No
01	Pipe type	06	08	Stiffness Matrix	Table.1
				Global co-efficient matrix	Table.2
				Node voltage of each node	Table.3
02	Step type	24	22	Stiffness Matrix	Table.4
				Global co-efficient matrix	Table.5
				Node voltage of each node	Table.6
03	Taper type	22	20	Stiffness Matrix	Table.7
				Global co-efficient matrix	Table.8
				Node voltage of each node	Table.9
04	Bend type	24	21	Stiffness Matrix	Table.10
				Global co-efficient matrix	Table.11

				Node voltage of each node	Table.12

Same procedure can be used to obtain stiffness, global co-efficient matrix and node voltages using four node triangles or five node triangles.

CONCLUSION

In the paper we had found out the stiffness matrix of each element, global co-efficient matrix of each model along with the node voltage of each node of waveguide models. After finding out all the above, the results are tabulated. The Finite element method is very easy and effective method to find all matrix and node voltages with triangular element with three nodes.

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